Working with Functions and Inverses

Solutions

(1)

(a) $f(x) = 4x + 7$

one-to-one (set $f(x_1) = f(x_2)$)

$4x_1 + 7 = 4x_2 + 7$

$4x_1 = 4x_2$

$x_1 = x_2$ Invertible ✓

Inverse

$x = 4y + 7$

$x - 7 = 4y$

$y = \frac{x-7}{4}$

$f^{-1}(x) = \frac{x-7}{4}$

$\text{Dom}(f) = \mathbb{R} = \text{Ran}(f^{-1})$

$\text{Ran}(f) = \mathbb{R} = \text{Dom}(f^{-1})$

(b) $f(x) = \frac{x-2}{x+4}$

Inverse

$x = \frac{y-2}{y+4}$

$(y+4)x = y-2$

$yx+4x = y-2$

$yx-y = -2-4x$

$y(x-1) = -2-4x$

$y = \frac{-2-4x}{x-1} = \frac{2+4x}{1-x}$

$f^{-1}(x) = \frac{2+4x}{1-x}$

one-to-one

$\frac{x_1-2}{x_1+4} = \frac{x_2-2}{x_2+4}$

$(x_1-2)(x_2+4) = (x_2-2)(x_1+4)$

$x_1x_2 + 4x_1 - 2x_2 - 8 = x_1x_2 + 4x_2 - 2x_1 - 8$

$6x_1 = 6x_2$

$x_1 = x_2$ ✓ Invertible

$Dom(f) = \mathbb{R} \setminus -4 = Ran(f^{-1})$

$Dom(f^{-1}) = \mathbb{R} \setminus 1 = Ran(f)$

(c) $f(x) = \log_2(x+1)$

<u>one-to-one</u>

$\log_2(x_1+1) = \log_2(x_2+1)$

$x_1+1 = x_2+1$

$x_1 = x_2$ ✓ Invertible

<u>Inverse</u>

$x = \log_2(y+1)$

$2^x = y+1$

$y = 2^x - 1$

$f^{-1}(x) = 2^x - 1$

$Dom(f) = (-1, \infty) = Ran(f^{-1})$

$Ran(f) = \mathbb{R} = Dom(f^{-1})$

(d) $g(x) = |x| - |x-6| = \begin{cases} 6 & x > 6 \\ 2x-6 & 0 \le x \le 6 \\ -6 & x < 0 \end{cases}$

Clearly not invertible because

$g(8) = 6 = g(10)$.

$\text{Dom}(g) = \mathbb{R}$

$\text{Ran}(g) = [-6, 6]$

(e) $h(x) = 10^{x/3}$

one-to-one

$10^{x_1/3} = 10^{x_2/3}$

$x_1/3 = x_2/3$

$x_1 = x_2$ ✓ Invertible

Inverse

$x = 10^{y/3}$

$\log(x) = y/3$

$y = 3\log(x)$

$h^{-1}(x) = 3\log(x)$

$\text{Dom}(h) = \mathbb{R} = \text{Ran}(h^{-1})$

$\text{Ran}(h) = (0, \infty) = \text{Dom}(h^{-1})$

(2) $f(x) = x^3 + 2$, $g(x) = \log(2x)$, $h(x) = 10^{x/2}$

(a) $f \circ g(x) = f(g(x)) = f(\log(2x)) = (\log(2x))^3 + 2$

(b) $g \circ f(x) = g(f(x)) = g(x^3+2) = \log(2(x^3+2))$

(c) $g \circ h(x) = g(h(x)) = g(10^{x/2}) = \log(2 \cdot 10^{x/2})$

(d) $h \circ g(x) = h(g(x)) = h(\log(2x)) = 10^{(\log(2x))/2}$

(e) $h \circ f \circ g(x) = h(f(g(x))) = h(f(\log(2x))) = h((\log(2x))^3 + 2)$

$$= 10^{\frac{(\log(2x))^3 + 2}{2}}$$

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(3) (a) $f(x) = 500 + 80x$. \$ as a fcn of hours

(b) $f^{-1}(x) = \frac{x - 500}{80}$. hours as a fcn of \$

(c) $f^{-1}(1220) = \frac{1220 - 500}{80} = \frac{720}{80} = 9$

If he/she made \$1220
then he/she worked 9 hours.